Vogel Approximation Method (VAM)

VAM is an improved version of the least cost method that generally produces better solutions. The steps involved in this method are:

Step 1: For each row (column) with strictly positive capacity (requirement), determine a **penalty** by subtracting the **smallest** unit cost element in the row (column) from the next **smallest** unit cost element in the same row (column).

Step 2: Identify the row or column with the **largest penalty** among all the rows and columns. If the penalties corresponding to two or more rows or columns are equal we select the topmost row and the extreme left column.

Step 3: We select X_{ij} as a basic variable if C_{ij} is the **minimum cost** in the row or column with **largest penalty**. We choose the numerical value of X_{ij} as high as possible subject to the row and the column constraints. Depending upon whether a_i or b_j is the smaller of the two i^{th} row or j^{th} column is crossed out.

Step 4: The Step 2 is now performed on the uncrossed-out rows and columns until all the basic variables have been satisfied.

Example

Consider the following transportation problem

Origin	Destination				ai
	1	2	3	4	
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
b_j	60	40	30	110	240

Note: a_i=capacity (supply)
b_j=requirement (demand)

Now, compute the penalty for various rows and columns which is shown in the following table:

Origin	Destination				a_i	Column Penalty
	1	2	3	4		
1	20	22	17	4	120	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
b_j	60	40	30	110	240	
Row Penalty	4	15	8	3		

Look for the highest penalty in the row or column, the highest penalty occurs in the **second column** and the minimum unit cost i.e. c_{ij} in this column is c_{12}=22. Hence assign 40 to this cell i.e. x_{12}=40 and cross out the second column (since second column was satisfied). This is shown in the following table:

Origin	Destination				a_i	Column Penalty
	1	2	3	4		
1	20	22 **40**	17	4	80	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
b_j	60	40	30	110	240	
Row Penalty	4	15	8	3		

The next highest penalty in the uncrossed-out rows and columns is 13 which occur in the **first row** and the minimum unit cost in this row is c_{14}=4, hence x_{14}=80 and cross out the first row. The modified table is as follows:

Origin	Destination				ai	Column Penalty
	1	2	3	4		
1	~~20~~	~~22~~ **40**	~~17~~	~~4~~ **80**	0	13
2	24	37	9	7	70	2
3	32	37	20	15	50	5
bj	60	40	30	110	240	
Row Penalty	4	15	8	3		

The next highest penalty in the uncrossed-out rows and columns is 8 which occurs in the **third column** and the **minimum cost** in this column is c_{23}=9, hence x_{23}=30 and cross out the third column with adjusted capacity, requirement and penalty values.
The modified table is as follows:

Origin	Destination				a_i	Column Penalty
	1	2	3	4		
1	20	22 **40**	17	4 **80**	0	13
2	24	37	9 **30**	7	40	17
3	32	37	20	15	50	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

The next highest penalty in the uncrossed-out rows and columns is 17 which occurs in the **second row** and the **smallest cost** in this row is c_{24}=15, hence x_{24}=30 and cross out the fourth column with the adjusted capacity, requirement and penalty values. The modified table is as follows:

Origin	Destination				a_i	Column Penalty
	1	2	3	4		
1	20	22 **40**	17	4 **80**	0	13
2	24	37	9 **30**	7 **30**	10	17
3	32	37	20	15	50	17
bj	60	40	30	110	240	
Row Penalty	8	15	8	8		

The next highest penalty in the uncrossed-out rows and columns is 17 which occurs in the **second row** and the **smallest cost** in this row is c_{21}=24, hence xi_{21}=10 and cross out the second row with the adjusted capacity, requirement and penalty values.
The modified table is as follows:

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20	22 **40**	17	4 **80**	0	13
2	24 **10**	37	9 **30**	7 **30**	0	17
3	32	37	20	15	50	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

The next highest penalty in the uncrossed-out rows and columns is 17 which occurs in the **third row** and the **smallest cost** in this row is c_{31}=32, hence xi_{31}=50 and cross out the third row or first column.
The modified table is as follows:

Origin	Destination 1	2	3	4	a_i	Column Penalty
1	20 	22 **40**	17	4 **80**	0	13
2	24 **10**	37	9 **30**	7 **30**	0	17
3	32 **50**	37	20	15	0	17
b_j	60	40	30	110	240	
Row Penalty	8	15	8	8		

The transportation cost corresponding to this choice of basic variables is

22*40+4*80+9*30+7*30+24*10+32*50=3520